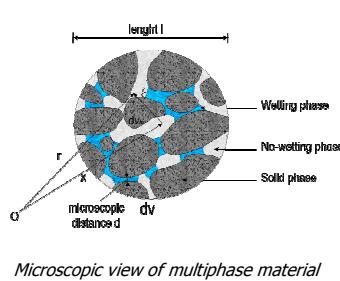


# A model for thermo-hydro-mechanical analysis of multiphase porous media in dynamics

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**Targets of the research:** This work presents the development of a mathematical and numerical model for the analysis of thermo-hydro-mechanical behavior of multiphase porous materials in dynamics [1], and its implementation in the finite element code COMES-GEO developed at University of Padua [2, 3].  
In this model, soils are modeled as non-isothermal multiphase porous media, where interstitial voids of the deforming solid matrix may be filled with liquid water, water vapor and dry air or other gas. To handle this multiphase system, an analytical multi-scale approach has been used by the general frame of averaging theories. Phase changes of water (evaporation-condensation, adsorption-desorption) and heat transfer through conduction and convection, as well as latent heat transfer are considered.  
The u-p-T formulation is advocated and the governing equation are [1]:



$$\operatorname{div} \boldsymbol{\sigma} + \rho (\mathbf{g} - \mathbf{a}^s) = 0$$

Linear momentum balance equation of the mixture

$$\begin{aligned} & \left( \rho C_p \right)_{\text{eff}} \frac{\partial T}{\partial t} - \operatorname{div} (\chi_{\text{eff}} \operatorname{grad} T) - \Delta H_{\text{vap}} \rho^w S_w \operatorname{div} \mathbf{v}^s - \Delta H_{\text{vap}} \rho^w \frac{n S_w \partial p^w}{K_w \partial t} + \Delta H_{\text{vap}} \beta_{sw} \frac{\partial T}{\partial t} \\ & + \left[ \rho_w C_p n S_w \frac{k^w \mathbf{k}}{\mu^w} (-\operatorname{grad} p^w + \rho^w \mathbf{g}) \right] \cdot \operatorname{grad} T - \Delta H_{\text{vap}} \rho^w (\rho^w - \rho^{gw}) \frac{\partial S_w}{\partial t} \\ & + \rho_g C_p n S_g \frac{k^{rg} \mathbf{k}}{\mu^s} (-\operatorname{grad} p^s + \rho^s \mathbf{g}) \\ & - \operatorname{div} \left[ \rho^w \frac{k^w \mathbf{k}}{\mu^w} (-\operatorname{grad} p^w + \rho^w \mathbf{g}) \right] \Delta H_{\text{vap}} = 0 \end{aligned}$$

Energy balance equation of the mixture

$$\begin{aligned} & \rho^w \frac{n S_w \partial p^w}{K_w \partial t} + [\rho^w S_w + \rho^{gw} S_g] \chi \operatorname{div} \mathbf{v}^s - \beta_{sw} \frac{\partial T}{\partial t} + n(\rho^w - \rho^{gw}) \frac{\partial S_w}{\partial t} \\ & + n S_g \frac{\partial \rho^{gw}}{\partial t} + \operatorname{div} \mathbf{J}_g^{gw} + \operatorname{div} \left[ \rho^w \frac{k^w \mathbf{k}}{\mu^w} (-\operatorname{grad} p^w + \rho^w \mathbf{g}) \right] \\ & + \operatorname{div} \left[ \rho^{gw} \frac{k^{rg} \mathbf{k}}{\mu^s} (-\operatorname{grad} p^s + \rho^s \mathbf{g}) \right] = 0 \end{aligned}$$

Water species mass balance equation

$$S_g \operatorname{div} \mathbf{v}^s + \frac{n S_g \partial p^{ga}}{\rho^{ga} \partial t} + \frac{1}{\rho^{ga}} \operatorname{div} \mathbf{J}_g^{ga} - n \frac{\partial S_w}{\partial t}$$

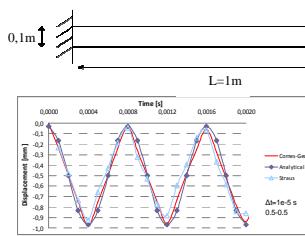
$$+ \frac{1}{\rho^{ga}} \operatorname{div} \left[ \rho^{ga} \frac{k^{rg} \mathbf{k}}{\mu^s} (-\operatorname{grad} p^s + \rho^s \mathbf{g}) \right] + n \beta_s S_g \frac{\partial T}{\partial t} = 0$$

Dry air mass balance equation

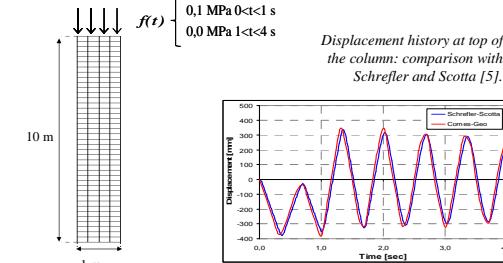
**Obtained results:** The formulation and the implemented solution procedure are validated through the comparison with literature benchmarks, finite element solutions or analytical solutions when available. The finite element results are plotted in the following pictures.

## Isothermal single phase solids model:

### 1. Wave propagation in a solid bar (Sluys, 1992 [4])

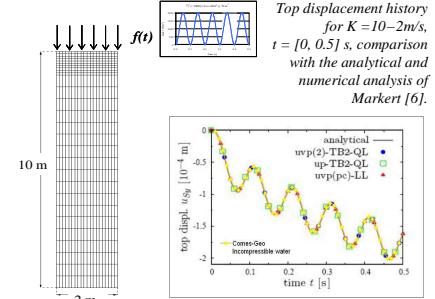


### 2. Wave propagation problem in a dry soil column [5]



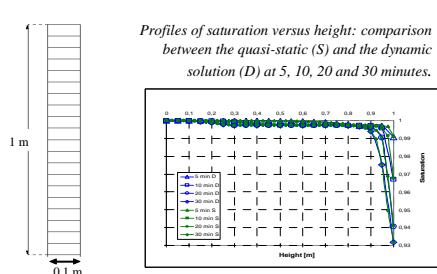
## Isothermal water saturated model:

### Dynamic consolidation of a saturated poroelastic column under harmonic load [6]

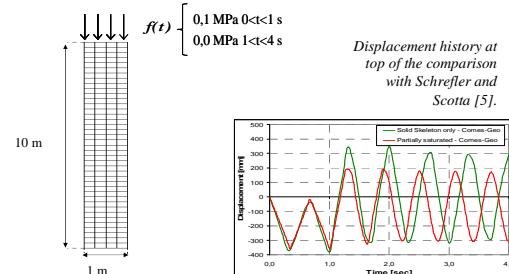


## Validation of the isothermal two phase flow model:

### 1. Quasi-static drainage (Liakopoulos test) [9]

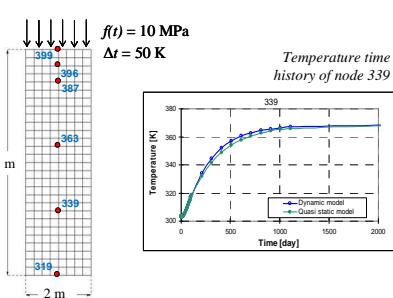


### 2. Sand column subjected to a step load [5]



## Non-isothermal water saturated model:

### Quasi-static mechanical consolidation problem (Aboustitt test)



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[4] L. J. Sluys (1992). Wave propagation, localisation and dispersion in softening solids. Ph.D. Thesis Technische Univ., Delft (Netherlands). Dept. of Civil Engineering.

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